Benjamin SPECTOR
Laboratoire de linguistique formelle, Paris 7/Ecole Normale Supérieure
benjamin.spector@ens.fr

Exhaustive interpretations: what to say and what not to say

LSA Institute, workshop on "Context and Content", Cambridge, July 15th, 2005.

Two questions about questions and answers

(1) a. Among Peter, Mary, and Jack, who came ?
b. Peter <came>

→ Peter came, neither Mary nor Jack did

- Why does the speaker only give explicitly his *positive* information ?

- Why does the hearer infer that only Peter came?

(2) a. Not Mary
b. Mary didn’t come

→ Nothing is inferred regarding Peter and Jack, apart from the fact that either the speaker does not know much about them, or takes them to be irrelevant.

(3) Mary, and not Jack

→ The speaker either does not know about Peter, or does not take him into consideration.

(4) –Among the chimists, the linguists and the philosophers, who came ?

(5) Some philosophers → No non- philosopher came

(6) No philosopher
→ The speaker does not know much about non-philosophers

(7) Some philosophers and no linguist
→ The speaker does not know much about chemists

(8) Between three and five philosophers
→ No non-philosopher came

Exhaustive readings as gricean inferences
(Spector 2003, Van Rooy & Schulz 2004, 2005):

The hearer's inferences are logical consequences of the following assumptions :

a. Grice's maxims of *quantity* and *quality*:
The speaker's answer is the most informative positive answer that he considers true

b. *Competence assumption*:
The speaker is as informed as is possible (relatively to what the question is about) given a.

What is a (positive) answer ?

Let D be a mutually known finite domain of quantification. Let Q be a question of the form ‘?xP(x)’.

- An *answer* is a member of the boolean closure of $\{P(d_1),\ldots,P(d_n)\}$, with $\{d_1,\ldots, d_n\}$ being a set of rigid constants s.t. every member of D is named by some constant d_i.

- A *positive answer* is a member of the closure under $\wedge$ and $\vee$ of $\{P(d_1),\ldots,P(d_n)\}$

G & S

In a world w, the **most informative answer** to [[?xP(x)]] is :

$\lambda w' (\forall d, (d \in P(w) \rightarrow d \in P(w')) \wedge (d \notin P(w) \rightarrow d \notin P(w'))$

i.e:

$\lambda w'(P(w') = P(w))$

Hence:
$[[?x\ P(x)]] = \lambda w.\lambda w'(P(w') = P(w))$

The complete answer to $[[?xP(x)]]$ in w is simply $[[?xP(x)]](w)$, i.e. $\lambda w'.(P(w') = P(w))$

i.e. the *complete answer* to the question in a world w is the proposition that states, for any d such that P(d) is true in w, *that* P(d) is true, and for any d such that P(d) is false in w, *that* P(d) is false.

Answerhood again

Notation: $w \cong_Q w'$, alternatively $w \cong_P w'$ iff $w' \in GS\text{-}Q(w)$, i.e. $P(w) = P(w')$.

A proposition S is an answer (is *strongly relevant*) *if* :

a) S rules out at least one complete answer:
$\exists w\ (A \cap GS\text{-}Q(w)) = \emptyset$

b) S does not distinguish between two worlds in which the extension of P is the same:
$\forall w \forall w'\ (w \cong_Q w' \rightarrow (w \in A \leftrightarrow w' \in A))$

Fact: A is strongly relevant iff A is a non-tautological member of the boolean closure of $\{P(d_1),\ldots, P(d_n)\}$

Positivity again

Fact: A is a *positive answer* iff:

$\forall w \in A\ \forall w'\ (P(w) \subseteq P(w') \rightarrow w' \in A\}$

Notation : $w \leq_P w'$ iff $P(w) \subseteq P(w')$

Definition: A proposition A is P-positive if:
$\forall w \in A\ \forall w'\ (w \leq_P w') \rightarrow w' \in A$

G&S's version of *quantity*:

The speaker's answer must be the most informative answer that he deems true:

Let i be the speaker's information state.
$i/P = \{w: \exists w' \in i, w \cong_P w'\}$
i.e. $i/P = \cup_{w \in i} \{w': P(w) = P(w')\}$

Fact: i/P is the most informative answer entailed by i.
G&S's version of quantity: The speaker's answer is i/P

Quantity according to Spector (2003), Van Rooy & Schulz (2004):

The speaker's answer must entail (be ?) the strongest positive answer that he considers true

$Pos_P(i) = \{w: \exists\, w' \in i\ (w' \leq_P w)\}$, i.e.
$Pos_P(i) = \cup_{w \in i}\ \{w': w \leq_P w'\}$

Fact: $Pos_P(i)$ is the strongest P-positive proposition entailed by i

VR&S, Spector's version of *quality* and *quantity*:

If i is the speaker's information state and A is the speaker's answer to a question Q whose predicate is P, then :

$i \subseteq A \subseteq Pos_P(i)$, which is equivalent to
$i \subseteq A$ and $Pos_P(A) = Pos_P(i)$.

Quality, *Quantity* and *Competence* :

The speaker's information state i belongs to the following set:

$$\mathrm{Max}(A,P) = \{i: i \subseteq A \wedge \mathrm{Pos}_P(A) = \mathrm{Pos}_P(i) \wedge \forall i' ((i' \subseteq A \wedge \mathrm{Pos}_P(A) = \mathrm{Pos}_P(i')) \rightarrow \neg (i'/P \subset i/P))\}$$

Deriving exhaustivity:

$Exh(A,P) = \{w: w \in A \wedge \neg \exists w' \in A, w' <_P w\}$

Example :

"John or Mary came" D = {j,m,s}

j	m	j,m
j,s	m,s	j,m,s

Theorem:

Max(A, P) = {i: i/P = Exh(A,P)}

NB: this predicts that all answers are interpreted exhaustively, contrary to fact:
"Not Mary" does not implicate that nobody came.

G&S and the special role of positivity

G&S's *quantity* predicts that a speaker should give all his relevant information, including negative information, contrary to fact.

They solve this problem by postulating an exhaustivity operator, so that (1)b. actually literally *means* "Peter and nobody else came".

But the pragmatic account of exhaustivity crucially needs to assume that (1)b, in terms of its literal meaning, just means "Peter came".

Questions as partial pre-orders

GS:
$[[?xP(x)]] = \lambda w.\lambda w' (\forall d \in D, d \in P(w) \rightarrow d \in P(w')$ & $d \notin P(w) \rightarrow d \notin P(w'))$

My revision:
$[[?xP(x)]] = \lambda w.\lambda w' (\forall d \in D, d \in P(w) \rightarrow d \in P(w'))$
i.e: $= \lambda w.\lambda w' (w \leq_P w')$
→ The complete answer in a world w is *the strongest P-positive proposition that is true in w*.

Note that $Q(w)(w') = 1$ iff $w \leq_P w'$

Deriving Positive Quantity

Suppose your information state is $\{w_1, w_2\}$. The question asks for the strongest true positive P-proposition, that is, for $Q(w_1)$ if w_1 is the actual world, and $Q(w_2)$ if w_2 is the actualy world.
Since you don't know in which world you are, your answer will be '$Q(w_1)$ or $Q(w_2)$'.

More generally, if i is the speaker's information state, his answer A must entail:
$\cup_{w\in i}\{Q(w)\} = \cup_{w\in i}\{\lambda w': w \leq_P w'\} = \{w': \exists w \in i, w' \leq_P w\}$
$= Pos_P(i)$

Relationship between G&S and partial-order semantics (cf. also Heim 1994)

Fact: $w_1 \cong_P w_2$ iff $Q(w_1) = Q(w_2)$

That is: if two worlds w_1 and w_2 are such that the most informative true positive proposition is the same in both, then the extension of P is the same in both (and the other way around).

Def:
- S is the complete answer to Q in w if $S = Q(w)$
- S is a *potential complete answer* to Q if $\exists w\ S=Q(w)$.

Fact: A complete answer can be true in w without being the complete answer in w.

Example : "Mary came" is true in all worlds in which everybody came, yet is not the complete answer in such worlds.

G&S's interpretation as an *implicature*.

If you ask [[?xP(x)]] in w_0, you ask the hearer to bring you in a state in which you know $Q(w_0)$. It can be safely assumed that you also want to know whether or not your request is being satisfied, that is, whether the proposition uttered by the answer is in fact the complete answer in the actual world.

A fully cooperative and well-informed speaker should therefore not only express the strongest true proposition, but furthermore indicate that what he has expressed is in fact the complete answer.

Let Op_Q be an operator which, when it applies to a proposition S, *states* that S is the complete answer to Q:

$[[Op_Q\ S]] = \lambda w.(S = Q(w))$
$Op_Q = \lambda S.\ \lambda w.(S{=}Q(w)\}$

Let GS be an operator that turns a question into a function which, to each world w, associates the proposition $Op_Q(Q(w))$, i.e. the proposition that Q(w) is the complete answer:

$[[GS\ Q]](w) = Op_Q(Q(w)) = \lambda w'.(Q(w) = Q(w'))$
$= \lambda w'.(w \cong_P w')$

Exhaustivity in simple clothes

Suppose the speaker's answer is "Mary came". The questioner can assume, as a default, that the proposition "Mary came" is the strongest true P-positive proposition, i.e. that Mary and nobody else came.

Facts:

- If S is a potential complete answer to Q, then $Op_Q(S)$= Exh(A,P)

BUT

- If S is not a potential complete answer to Q, then $Op_Q(S)$ is the contradiction

Embedded questions

(9) Jack knows who came

G&S : "Jack knows who came" is true in w if Jack knows the proposition that is the complete answer to "who came" in w

G&S → For any x, if x came, Jack knows that x came, and if x didn't come, Jack knows that x didn't come

Partial-order → For any x, if x came, Jack knows that x came (= "weak exhaustivity")

An improvement

[[Jack knows ?xPx]](w) = 1 iff $Pos_P(j,w) = $ [[?xPx]](w)
with $Pos_P(j,w) = \{w: \exists\, w' \in Dox(j, w), w \leq_P w'\}$

[[know]] = $\lambda Q_{<s, st>}.\lambda x_{<e>}.\lambda w_{<s>}.\ Pos_Q(x, w) = Q(w)$.

→ For any x who came, Jack knows that x came, and for any x who didn't come, Jack does not have the belief that x came.

Partial-order semantics is NOT falsified by the fact that weak exhaustive readings maybe don't exist, since the G&S denotation of a question is definable in terms of its basic denotation.

"Jack knows Q" is true in w iff Jack knows in w that the complete answer to Q in w is Q(w), i.e :

[[know]] =

$\lambda Q_{<s,\ st>}.\lambda x_{<e>}.\lambda w_{<s>}.\ Dox(x, w) \subseteq \lambda w'(Q(w') = Q(w))$

i.e

$\lambda Q_{<s,\ st>}.\lambda x_{<e>}.\lambda w_{<s>}.\ Dox(x, w) \subseteq GS\text{-}Q(w)$

Does the weak exhaustive reading exist ?

Suppose A, B, C and D were invited to a certain party. A and B came, and neither C nor D came. Jack knows that A and B came, and is uncertain concerning C and D.

What about:

(10) Jack knows who came
(11) Jack doesn’t know who came

Replace “know” with “guess” :

(12) Jack guessed who came
(13) Jack didn’t guess who came

Negative answers

Positive Quantity predicts that if the speaker uses a negative answer, then he must have no positive belief (contrary to Zimmerman & Von Stechow 1984)

That's not enough:

(14) Less than three chemists came

We also infer that the speaker has no more negative knowledge that what he explicitly expressed: he does not have the belief that, say, philosophers didn't come, nor that there were in fact less than *two* chemists

Potential solution:

- If the answer A is negative, then A must be the most informative negative proposition that the speaker deems true.

What about implicatures in DE contexts ?

(15) Less than three chemists came

We tend to infer that one chemist or two must have come (contrary to what I predict)

Note that implicatures in DE contexts are much weaker than those arising in UE ones:

(16) Jack read 5 books
>> exactly 5
(17) Jack didn't read 5 books
*>> Jack read exactly 4 books

Non-monotonic answers

(18) a. Between two and five chemists
b. Some philosophers and between two and five chemists

→ *Exh* yields :

a. Exactly two chemists and nobody else
b. Some philosophers but not all, exactly two chemists, and nobody else

(19) a. Peter and not Mary
b. Some philosophers and no chemist

→ Not all the philosophers (and the speaker does not know much about linguists)

Exh yields:

a. Peter and nobody else
b. Some but not all of the philosophers and no non-philosopher

Two additional principles (informally)

Negative quantity (2nd version)

The speaker's answer must contain all the negative information he has regarding individuals he chooses to talk about negatively

→ accounts for the readings of (18)

(18) a. Between two and five chemists
b. Some philosophers and between two and five chemists

Symmetry principle:

If the speaker has only negative knowledge regarding two individuals d and d’, then his answer must treat them on a par, i.e. either mention both of them, or non of them.

→ accounts for the absence of exhaustivity effects with negative answers ((6) & (14)), as well as (19).

(19) a. Peter and not Mary
b. Some philosophers and no chemist

Aboutness

1. A proposition A positively P-concerns an individual d if there is a world w such that $w \in A$, $d \in P(w)$, and the w' identical to w except that $d \notin P(w')$ is such that $w' \notin A$.

2. A proposition A negatively P-concerns an individual d if there is a world w such that $w \in A$, $d \notin P(w)$, and the world w' identical to w except that $d \in P(w')$ is such that $w' \notin A$.

3. A proposition A P-concerns an individuals d if A positively P-concerns d or negatively P-concerns d.

Fact

Let A be a strongly P-relevant proposition.

-Let A+ be the set of individuals that A positively P-concerns.

$A+ = \{d_1,...,d_n\}$

- Let A- be the set of individuals that A negatively P-concerns

$A- = \{e_1,...,e_m\}$

Then A belongs to the closure under $\wedge$ et $\vee$ of $E = \{P(d_1),...., P(d_n), \neg P(e_1),..., \neg P(e_m\}$, and does not belong to the closure under $\wedge$ and $\vee$ of any proper subset of E.

Illustration

(20) Three linguists came

(21) Less than four linguists came

(22) Many linguists and few chemists came

(23) Between three and five linguists came

Fact:

Let Q be a question whose predicate is P. Then a strongly Q-relevant proposition S is P-positive if and only if S does not negatively P-concern any individual; and a Q-relevant proposition S is P-negative if and only if S does not positively P-concern any individual.

Symmetry principle (final version)

If i is the speaker’s information state, and A is his answer, then:

For any d, d’, if i negatively P-concerns both d and d’, and does not positively P-concerns either d or d’, then A must either negatively P-concerns both, or not concern any of them.

<u>Conflating negative and positive quantity</u>

Maxim of quantity (new version):
Let A be answer, with
A- = $\{e_{1,...}, e_n\}$ and D = $\{d_1,...,d_m\} \cup$ A-

Then the speaker's answer must be the most informative member of the closure under $\wedge$ and $\vee$ of :

$$\{P(d_1),..., P(d_m),P(e_1)...,P(e_n), \neg P(e_1), \neg P(e_2),...,\neg P(e_m)\}$$

i.e. : the speaker says everything relevant he knows about the e_i's, besides giving all his positive information.

With

$Pos^*_{A,P}(i) =$

$\cup_{w\in i} \{w': (P(w)\cap A-) = (P(w') \cap A-) \ \& \ P(w) \subseteq P(w')\}$

We end up : $A \subseteq Pos^*_{A,P}(i)$

Finally:

- $i\subseteq A\subseteq Pos_{A,P}(i)$
- For any d, d', if i negatively P-concerns both d and d', and does not positively P-concerns either d or d', then A must either negatively P-concerns both, or not concern any of them.

$w \leq_{P,A} w'$ iff $(P(w) \cap A\text{-}) = (P(w) \cap A\text{-})$ & $P(w) \subseteq P(w')$

$w <_{P,A} w'$ iff $w \leq_{P,A} w'$ & $\neg(w' \leq_{P,A} w')$

$Exh^*(A, P) = \{w \in A: \neg\exists\, w' \in A\ (w' <_{P,A} w))$

A= « Between one and two linguists »

A- = $\{l_1, l_2, l_3\}$ D = $\{l_1, l_2, l_3, c\}$

l_1	l_2	l_3	$l_1 l_2$	$l_1 l_3$	$l_2 l_3$
$l_1 c$	$l_2 c$	$l_3 c$	$l_1 l_2 c$	$l_1 l_3 c$	$l_2 l_3 c$

« Between one and two linguists or one philosopher »

$D = \{ l_1, l_2, l_3, p_1, p_2, c\}$ $A\text{-} = \{l_1, l_2, l_3\}$

l_1	l_2	l_3	l_1l_2	l_1l_3	l_2l_3	p_1	p_2	p_1p_2
l_1c	l_2c	l_3c	l_1l_2c	l_1l_3c	l_2l_3c	p_1c	p_2c	p_1p_2c
l_1p_1	$l_2\ p_1$	l_3p_1	$l_1l_2p_1$	$l_1l_3p_1$	$l_2l_3p_1$			
l_1p_2	l_2p_2	l_3p_2	$l_1l_2p_2$	$l_1l_3p_2$	$l_2l_3p_2$			
$l_1p_1\ p_2$	l_2p_1 p_2	l_3p_1 p_2	$l_1l_2p_1$ p_2	$l_1l_3p_1$ p_2	$l_2l_3p_1$ p_2			
l_1cp_1	l_2c p_1	l_3c p_1	l_1l_2c p_1	l_1l_3c p_1	$l_2l_3cp_1$			
l_1cp_2	l_2cp_2	l_3c p_2	l_1l_2c p_2	l_1l_3c p_2	l_2l_3c p_2			
l_1cp_1 p_2	l_2c $p_1\ p_2$	l_3c p_1 p_2	l_1l_2c $p_1\ p_2$	l_1l_3c $p_1\ p_2$	l_2l_3c $p_1\ p_2$			
						$p_1l_1l_2l_3$	$p_2l_1l_2l_3$	$p_1p_2l_1l_2l_3$
						$p_1l_1l_2l_3c$	$p_2l_1l_2l_3c$	$p_1p_2l_1l_2l_3c$

Quasi-positivity

A proposition A is quasi P-positive if there is no d such that A negatively P-concerns d and does not P-positively concerns d

Claim: The set of answers that give rise to an exhaustivity effect (i.e. trigger a negative inference with respect to individuals that they do not P-concern) is exactly the set of quasi-positive answers.

“and” vs. “but” and decreasing quantifiers

(24) a.Some philosophers **and** few chemists
?>> No linguist
b. Some philosophers **but** few chemists
*>> No linguist

Tentative account :

-« few » is ambiguous between a monotone-decreasing reading (*few*, *if any*), and a non-monotonic one (*at least one but few*).

- coordination with *but* requires the two DPs to be of opposite monotonicity directions

Predictions

The following answers should not allow exhaustive readings:

(25) Some philosophers and few chemists, **if any**

(26) Some philosophers and few chemists who got ***any*** interesting results

SELECTED REFERENCES

Groenendijk, J. & Stokhof, M.: 1984, *Studies in the Semantics of Questions and the Pragmatics of Answers*, PhD Thesis, University of Amsterdam.

Heim, I.:1994, "Interrogative semantics and Kartunnen's semantics for know", *Proceedings of the 9th Annual Conference (Ben Gurion University of the Negev 1993) and of the workshop on discourse (Hebrew University of Jerusalem, 1993)*

Spector, B.: 2003, "Scalar implicatures: exhaustivity and Gricean Reasoning", in B. ten Caten (ed.), *Proceedings of the ESSLLI 2003 student session*, Vienna

Van Rooy, R, & Schulz, K.: 2004, "Exhaustive interpretation of complex sentences", *Journal for Logic, Language and Information*.

Stechow, A. von & Zimmermann, T. E.: 1984, "term answers and contextual change", *Linguistics,* **22**, 3-40